Sadiq Public School

Do the right, fear no man

Subject: Mathematics **Class: I1** **Day: SATURDAY (16/11/ 2024)**

REVISION

Lesson [Chapter # 10] This lesson is about introduction of double angle, half angle and triple angle identities.

Information:

- **You can review this topic from your text book pages (from page # 328 and 333)**
- **You can get help from the following link:** **https://youtu.be/8ZX_9uqqkTQ?si=6ooZ6GzvM9sYbv_x**
- https://youtu.be/8KTkJTuMfPI?si=ZdF_uBX1-SR0cbC8
- https://youtu.be/nKKLAcu5bHc?si=dJbviUWb4TOt19Zo **for further understanding of exercise 10.3.**

Double Angle Identities

Dear students

We have studied the basic trigonometric identities such as $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

These formulas will be used to obtain the double angle identities as

Putting $\beta = \alpha \; in \; \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta, we \; get \; \sin(\alpha+\alpha) = \sin\alpha\cos\alpha + \cos\alpha\sin\alpha$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

Now putting $\beta = \alpha \; in \; \cos(\alpha+\beta) = cos\alpha\cos\beta + \sin\alpha\sin\beta, we \; get \; \cos(\alpha+\alpha) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\text{Cos } 2\alpha = \cos^2\alpha - \sin^2\alpha$

$= 2\cos^2\alpha - 1$

$= 1 - 2\sin^2\alpha$

Half Angle Identities: $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$

$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

Triple Angle Identities: $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

Similarly other identities can be proved. A link has been provided to understand proof of all half angle and triple angle identities. Another link has also been provided for the help to solve next questions of the exercise.

Exercise 10.3

Q#9: LHS = $\frac{2\sin\theta\ \sin 2\theta}{\cos\theta + \cos 3\theta} = \frac{2\sin\theta\ \sin 2\theta}{\cos\theta + 4\cos^3\theta - 3\cos\theta} = \frac{2\sin\theta\ \sin 2\theta}{4\cos^3\theta - 2\cos\theta} = \frac{2\sin\theta\ \sin 2\theta}{2\cos\theta(2\cos^2\theta - 1)}$

$= \frac{\sin\theta\ \sin 2\theta}{\cos\theta \times \cos 2\theta} = \frac{\sin 2\theta}{\cos 2\theta} \times \frac{\sin\theta}{\cos\theta} = \tan 2\theta . \tan\theta = RHS$

Q#13: LHS = $\frac{\sin 3\theta}{\cos\theta} + \frac{\cos 3\theta}{\sin\theta} = \frac{\sin 3\theta\ \sin\theta + \cos 3\theta . \cos\theta}{\sin\theta . \cos\theta} = \frac{\cos 3\theta . \cos\theta + \sin 3\theta\ \sin\theta}{\sin\theta . \cos\theta}$

$= \frac{\cos(3\theta - \theta)}{\sin\theta . \cos\theta} = \frac{\cos(2\theta)}{\sin\theta . \cos\theta} = \frac{2\cos(2\theta)}{2\sin\theta . \cos\theta} = \frac{2\cos(2\theta)}{\sin 2\theta .} = 2\cot 2\theta = RHS$

Example 3: Reduce $\cos^4\theta$ to an expression involving only function of multiples of θ raised to first power.

Solution: Consider $\cos^4\theta = (\cos^2\theta)^2 = (\frac{1+cos2\theta}{2})^2$ (using half angle identities)

$= \frac{(1+cos2\theta)^2}{2^2} = \frac{1}{4}((1)^2 + (cos2\theta)^2 + 2cos2\theta)$

$= \frac{1}{4}(1 + cos^2 2\theta + 2cos2\theta)$ (using half angle identity)

$= \frac{1}{4}(1 + (\frac{1+cos4\theta}{2}) + 2cos2\theta)$

$= \frac{1}{4}(\frac{2+1+cos4\theta+4cos2\theta}{2})$

$= \frac{1}{8}(3 + cos4\theta + 4cos2\theta)$

Synthesizing:

- You can review this topic from page 328 -332 from your textbook.
- Create notes in your Notebook after referring to the given text and the formulas in your textbook.

Practising: Revise all questions of exercise 10.3 (page 332) can be solved by using the above-mentioned identities. The link **https://youtu.be/nKKLAcu5bHc?si=dJbviUWb4TOt19Zo** **can be used for further understanding of exercise 10.3.**

B) Assessment:

Review the lessons of this week and revise the questions of previous exercise

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