Sadiq Public School

Do the right, fear no man

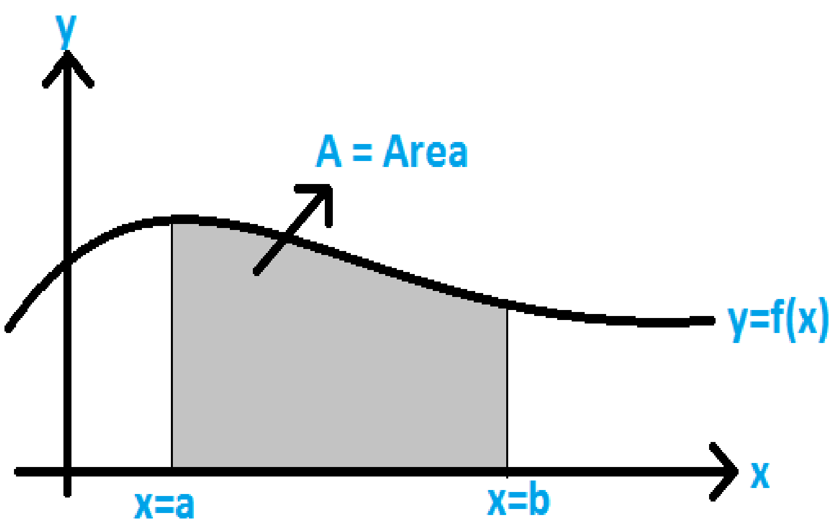
Subject: Mathematics **Class: I2** **Day: Saturday (16/11/24)**

Lesson: Integration (Chapter#3)

Area under the Curve Formula

The area under a curve between two points is found out by doing a definite integral between the two points. To find the area under the curve y = f(x) between x = a & x = b, integrate y = f(x) between the limits of a and b.

This area can be calculated using integration with given limits.



The formula for Area under the Curve = $\int_a^b$ **f(x)dx**

Solved Example

Question : Calculate the area under the curve of a function, f(x) = 7 – x^2, the limit is given as x = -1 to 2.
Solution:

Given function is, f(x) = 7- x^2 and limit is x = -1 to 2

Solution:

Given function is, f(x) = 7- x^2 and limit is x = -1 to 2

$$Area = \int_{-1}^{2}(7 - x^2)dx$$

$$= \left(7x - \frac{1}{3}x^3\right)\Big|_{-1}^{2}$$

$$= \left[7.2 - \frac{1}{3}(8)\right] - \left[7(-1) - \frac{1}{3}(-1)\right]$$

= [(42 – 8)/3] – [(1 – 21)/3]

= (34 + 20)/3

= 54/3

= 18 sq.units

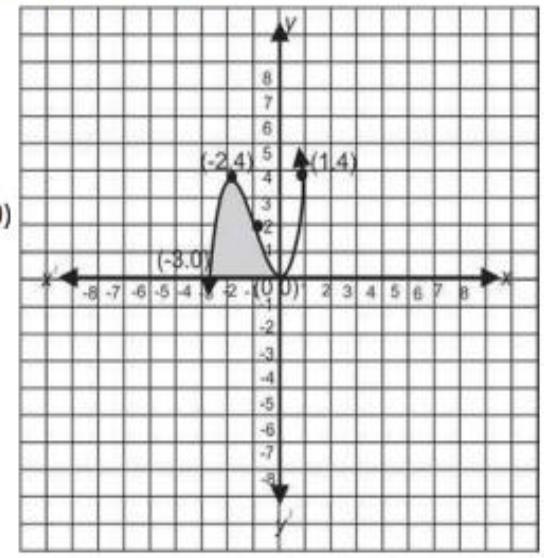
Example 2. **Find the area bounded by the curve $y = x^3 + 3x^2$ and the x-axis.**

Solution: Putting $y = 0$, we have

$$x^3 + 3x^2 = 0$$

$$\Rightarrow x^2 (x + 3) = 0 \Rightarrow x = 0, x = -3$$

The curve cuts the x-axis at (–3, 0) and (0, 0) (see the figure).



Thus the required area $= \int_{-3}^{0} (x^3 + 3x^2)\, dx$

$$= \left[\frac{x^4}{4} + x^3\right]_{-3}^{0}$$

$$= \left(\frac{0}{4} + 0\right) - \left(\frac{(-3)^4}{4} + (-3)^3\right)$$

$$= 0 - \left(\frac{81}{4} - 27\right) = -\left(\frac{81-108}{4}\right) = -\left(-\frac{27}{4}\right) = \frac{27}{4}$$

Example 3. **Find the area bounded by $y = x(x^2 - 4)$ and the x-axis.**

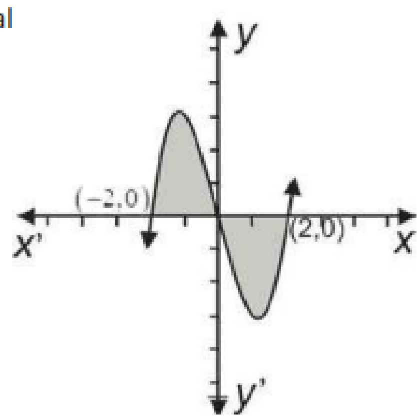
Solution: Putting $y = 0$, we have

$$x(x^2 - 4) \Rightarrow x = 0, x = \pm 2$$

The curve cuts the x-axis at (–2, 0), (0, 0) and (2, 0). The graph of f is shown in the figure and we have to calculate the area of the shaded region.

$$f(x) = x(x^2 - 4),$$

$f(x) \geq 0$ for $-2 \leq x \leq 0$, that is, the area in the interval [–2, 0] is above the x-axis and is equal to



$$\int_{-2}^{0} x(x^2 - 4)\, dx$$

$$= \int_{-2}^{0} (x^3 - 4x)\, dx = \left|\frac{x^4}{4} - 4\left(\frac{x^2}{2}\right)\right|_{-2}^{0} = \left[\frac{x^4}{4} - 2x^2\right]_{-2}^{0}$$

$$= 0 - \left(\frac{(-2)^4}{4} - 2(-2)^2\right) = 0 - \left(\frac{16}{4} - 8\right) = -(4 - 8) = 4$$

$f(x) \leq 0$ for $0 \leq x \leq 2$, that is, the area in the interval [0, 2] is below the x-axis and is equal to $-\int_{0}^{2} (x^3 - 4)\, dx = -\left[\frac{x^4}{4} - 2x^2\right]_{0}^{2}$

$$= -\left[\left(\frac{16}{4} - 2(4)\right) - 0\right]$$

$$= -[\,4 - 8\,] = -(-4) = 4$$

Thus the area of the shaded region = 4 + 4 = 8

- ➢ You can review this topic from page 53-56 from your textbook.
- ➢ Create notes in your Notebook after referring to the given text and the formulas in your textbook.
- ➢ Solve the following exam style questions to assess the extent of your learning.

Solve Q (1 to 13) of Exercise 3.7