Sadiq Public School

Do the right, fear no man

Subject: Mathematics **Class: H1** **Day: Saturday (16/11/24)**

Lesson: Practical maximum and minimum problems

A) Inquiry.

"Have you ever wondered how engineers determine the strongest shape for a bridge or how farmers maximize the area for their crops with limited fencing?" "Imagine you're designing a rectangular garden against a wall and want the maximum area for a fixed length of fencing. What would be your first steps to figure this out?"

B) Information:

1. Derivative and its Role: We know that the derivative of a function shows how it changes, and finding where the derivative is zero helps identify maximum or minimum values.
2. Critical Points: These are where the derivative equals zero or does not exist, marking potential max/min values.
3. Second Derivative Test: Using the second derivative helps determine if the critical point is a max or min.
4. Real-World Application: Show how to set up a function that represents the problem's requirements (e.g., area as a function of width for the garden problem).

Example1

2. Locate and Classify the critical points of $y = x^3 - 3x^2 - 9x + 1$.

$y = x^3 - 3x^2 - 9x + 1$

$y' = 3x^2 - 6x - 9$ *(at critical points y'= 0)*

$0 = 3x^2 - 6x - 9$ (divide both sides by 3)

$0 = x^2 - 2x - 3$

$(x - 3)(x + 1) = 0$

$(x = 3)$ and $(x = -1)$

If $x = 3, y = ?$

$y = x^3 - 3x^2 - 9x + 1$

$y = 3^3 - 3(3)^2 - 9(3) + 1$

$y = 27 - 27 - 27 + 1$

$y = -26$

If $x = -1, y = ?$

$y = x^3 - 3x^2 - 9x + 1$

$y = (-1)^3 - 3(-1)^2 - 9(-1) + 1$

$y = -1 - 3 + 9 + 1$

$y = 6$

critical points (3, -26) and (-1, 6)

Using first derivative test

At point (3, -26) let x = 2 and x = 4

at x = 2	at x = 4
$y' = 3x^2 - 6x - 9$	$y' = 3x^2 - 6x - 9$
$y' = 3(2)^2 - 6(2) - 9$	$y' = 3(4)^2 - 6(4) - 9$
$y' = 12 - 12 - 9$	$y' = 48 - 24 - 9$
$y' = -9$	$y' = 15$

Since y' changes from (–) to (+), point (3, -26) is **minimum point.**

At point (-1, 6) let x = -2 and x = 0

at x = -2	at x = 0
$y' = 3(-2)^2 - 6(-2) - 9$	$y' = 3(0)^2 - 6(0) - 9$
$y' = 12 + 12 - 9$	$y' = 12 + 12 - 9$
$y' = 15$	$y' = -9$

Since y' changes from (+) to (-), point (-1, 6) is **maximum point.**

Example 2

Example: If $f(x) = 3x^3 - 9x^2 - 27x + 15$ ***Find extreme values***

Solution: Step1: Find $f'(x)$

$$f'(x) = 9x^2 - 18x - 27$$

Step2: $f'(x) = 0$

$\therefore 9x^2 - 18x - 27 = 0$

$\therefore 9(x^2 - 2x - 3) = 0$

$\therefore 9(x + 1)(x - 3) = 0$

$\therefore \quad x = -1 \quad and \quad x = 3$

Step3: $f''(x) = 18x - 18$

Step4: Find $f''(x)$ and put values of x,

At $x = -1$, $f''(-1) = 18(-1) - 18 = -36 < 0$

Maximum value => $f(x) = 3(-1)^3 - 9(-1)^2 - 27(-1) + 15 = 30$

At $x = 3$, $f''(3) = 18(3) - 18 = +36 > 0$

Minimum value => $f(x) = 3(3)^3 - 9(3)^2 - 27(3) + 15 = -66$

C) Synthesizing:

1. Search from different sources about the above topic and record ideas.

2. Edit notes and combine concepts that are similar.

3. Synthesize by combining notes with what you already know about the topic.

4. Think about your new ideas and connect them to what you already know.

D) Practicing: Try to clear your basic concepts before moving ahead with the formulas. This will help you understand the sense of the formula. List down all formulas of this chapter from your text book on your notebook and whenever you are free, glance at them once. Solve Q # 7 to 15 of Exercise 8(C) but there is no need to send it to your subject teacher.